APPROXIMATE MODEL OF A HEAT EXCHANGER WITH DISTRIBUTED PARAMETERS

G. M. Faikin

Inzhenerno-Fizicheskii Zhurnal, Vol. 12, No. 4, pp. 491–495, 1967

UDC 536.27

The construction of an approximate model of a type of heat exchanger described by partial differential equations is discussed. A form of transfer function for this approximate model is proposed together with a calculation method and a block diagram for analog simulation purposes.

The approximate description of the transient processes in heat exchangers with distributed parameters is important in the analysis of the control process.

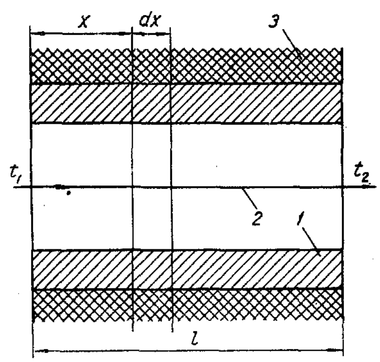

Fig. 1. Schematic of heat exchanger: 1) tube, 2) flow, 3) insulation.

The solutions of the systems of partial differential equations describing the heat-exchange process are quite complicated [1, 2]. Therefore the transfer functions cannot be set up directly on analog computers.

The purpose of this paper is to examine an approximation of the transfer function for an insulated tube (Fig. 1) through which a heat transfer agent flows at constant velocity.

We will find the approximation of the transfer function in the form [2]

$$W(p) = t_2(p)/t_1(p),$$

where $t_1(p)$ is the Laplace transform for the inlet temperature t_1 of the heat transfer agent, and $t_2(p)$ is the transform for the outlet temperature t_2.

In deriving the equations of dynamics the following assumptions are made:

1. The wall of the tube is treated as a lumped thermal capacitance.

2. The temperature of the wall surface in contact with the heat transfer agent is taken equal to some mean temperature.

3. Heat conduction along the axis of the tube is not taken into account either for the wall or for the flow.

4. The thermal insulation of the tube is assumed to be ideal.

Using the same terminology as in [2], we write the heat balance equations for an element dx of the heat transfer agent and the wall:

$$\frac{\partial t(x,\tau)}{\partial \tau} + w\frac{\partial t(x,\tau)}{\partial x} = k_1[v(x,\tau) - t(x,\tau)],$$

$$\frac{\partial v(x,\tau)}{\partial \tau} = k_2[t(x,\tau) - v(x,\tau)],$$

where $k_1 = \alpha_1 L_1/C_1\gamma_1 S_1$ is the constant coefficient of the heat balance equation for the heat transfer agent; $k_2 = \alpha_1 L_1/C_2\gamma_2 S_2$ is the constant coefficient of the heat balance equation for the tube wall.

The initial conditions are taken as the zero conditions. The boundary condition is

$$t(0,\tau) = t_1(\tau).$$

Several authors [1, 2] have solved this system of differential equations using the Laplace transformation.

The transfer function of the outlet temperature as a function of the inlet temperature for an insulated tube without allowance for heat conduction in the walls may be written as follows:

$$W(p) = \exp(-p\tau_0)\exp(-b_0)\exp\left(\frac{b_0}{1+Tp}\right),$$

where $\tau_0 = l/w$ is the transport lag; $T = 1/k$ the time constant of the tube, which does not depend on the tube length; and $b_0 = lk_1/w$ the "dimensionless length"—a quantity proportional to the length of the tube.

The amplitude-phase response for the given system has the following form:

$$W(j\omega) = a_1\exp(-j\varphi_1),$$

where

$$a_1 = \exp\left(\frac{b_0}{1+T^2\omega^2} - b_0\right),$$

$$\varphi_1 = -\omega\tau_0 - b_0\omega T/(1+\omega^2T^2).$$

Figure 2 shows the amplitude-frequency and phase-frequency responses of this system for several values of b_0; in this case $\tau_0 = 0$ (the additional phase shift due to this lag is $\tau_0\omega$), and the frequency is normalized to the time constant of the tube T.

The transient responses to stepwise variation of the inlet temperature for several values of b_0, shown in Fig. 3, were constructed by the method described in [3]. (The time axis is normalized to the time constant T.)

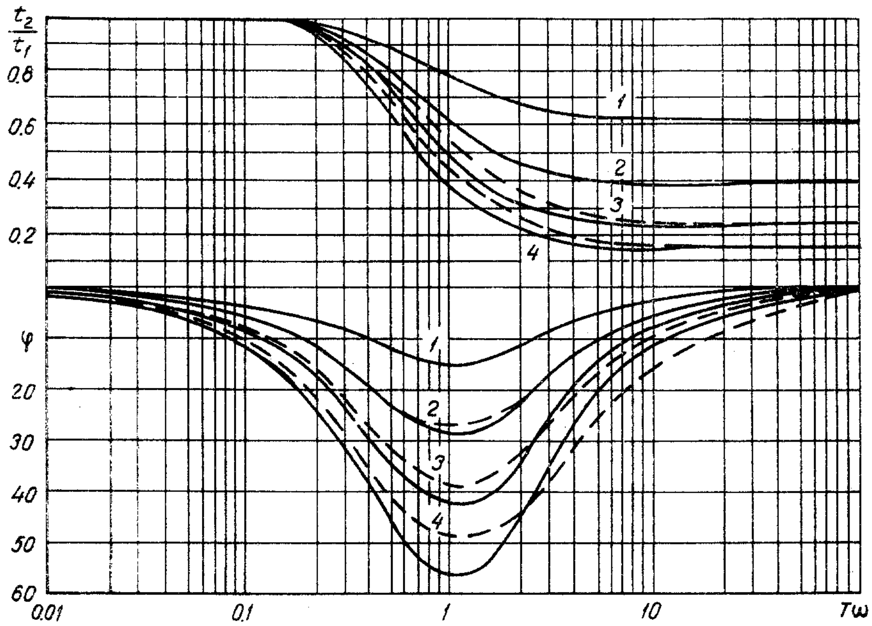

Fig. 2. Comparison of amplitude-frequency and phase-frequency responses for the heat transfer process in an insulated tube and its approximate model ($\tau_0 = 0$): 1, 2, 3, 4 for b_0 = 0.5, 1.0, 1.5 and 2.0; solid line is the exact solution; dashed line is the approximate solution.

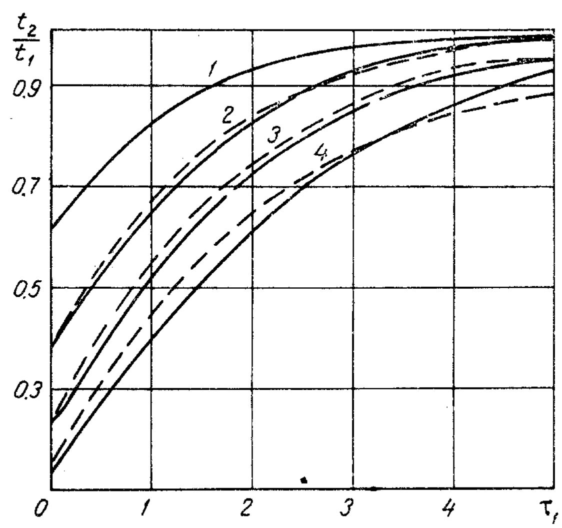

Fig. 3. Comparison of transient responses to stepwise variation of the inlet temperature for the heat transfer process in an insulated tube and its approximate model ($\tau_1 = (\tau - \tau_0)/T$). Solid line is the exact solution; dashed line is the approximate solution; 1–4, see Fig. 2.

It is interesting to note that the shape of the transient response curve does not depend on the liquid transport time, although it does depend on the flow velocity.

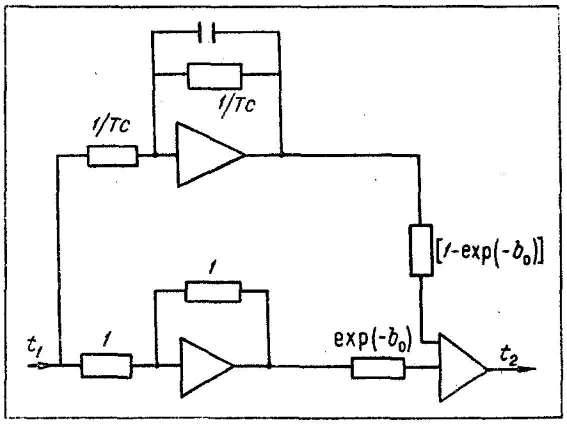

Fig. 4. Analog circuit for solving transfer function of the element determined by the heat transfer process.

In some approximation the actual physical process of heat transfer in an insulated tube can be represented as follows: following an abrupt increase in the temperature of the liquid entering at one end of the tube the temperature of the liquid at the tube outlet does not rise during the time required to transport the liquid along the length of the tube. As soon as transportation is complete, hot liquid, whose temperature has been reduced by the cold tube, flows out. Then, as each section of the tube is heated, the outlet temperature of the flow progressively rises to the steady state.

Thus, as may be seen from the transfer function itself and the description of the process, the transfer function W(p) may be represented as a pure delay element, determined by the transport lag of the heat transfer agent, connected in series with an element determined by the heat transfer process.

We will consider the heat transfer process in this system in detail, neglecting the temperature gradient of the tube in the direction of flow, i. e., lumping the thermal capacitance of the tube.

On the basis of our approximation of the process the heat balance equation for the heat transfer agent may be written as follows:

$$\frac{dt}{dx} = \frac{b_0}{l}(v - t).$$

Integrating this equation and considering that v does not depend on the coordinate x, we find the flow temperature at the tube outlet

$$t_2 = v + (t_1 - v)\exp(-b_0).$$

The heat balance equation for the wall is

$$T\frac{dv}{d\tau} = \frac{1}{b_0}(t_1 - t_2).$$

Solving this system of equations, and eliminating v, we obtain the transfer function of the heat transfer process in the following form:

$$W_1(p) = [\exp(-b_0)cTp + 1]/(cTp + 1),$$

where

$$c = b_0/[1 - \exp(-b_0)].$$

The analog circuit for solving the given transfer function is shown in Fig. 4. In order to set it up, it is necessary to know the values of the quantities T, b_0, and c.

Comparison of the characteristics presented in Figs. 2 and 3 shows quite good convergence of the exact and approximate transfer functions at values of $b_0 < 2$. In order to characterize the heat transfer system in an insulated tube with distributed parameters, in which the "dimensionless length" $b_0 > 2$, it is necessary to divide the part of the tube considered into individual sections, so that $b_0 < 2$, and calculate from the given values of b_0 and T for each section approximate transfer functions of the type considered, which are then combined.

Thus, the method described permits the operational approximation of transfer functions of a similar type, it being sufficient to determine the values of three quantities: the time constant of the tube T, the "dimensionless length" b_0, and the coefficient of the time constant c, which can easily be found from the physical and design parameters of the process.

NOTATION

$t(x, \tau)$ is the temperature of the heat transfer agent, °C; $v(x, \tau)$ is the wall temperature, °C; w is the flow velocity, m/sec; x is the coordinate of length, m; τ is the variable time, sec; k_1, k_2 are the constant coefficients of the heat balance equations for the flow and the wall, respectively, 1/sec; α_1 is the heat transfer coefficient, kcal/m^2 · sec · °C; L_1 is the inside diameter of tube, m; C_1 is the specific heat capacity of flow, kcal/kg · °C; γ_1 is the specific weight of flow, kg/m^3; S_1 is the internal cross section of tube, m^2; C_2 is the specific heat capacity of the wall, kcal/kg · · °C; γ_2 is the specific weight of the wall material, kg/m^3; S_2 is the cross section of the tube wall, m^2; τ_0 is the transport lag, sec; l is the length of section of tube, m; T is the time constant of tube, sec; b_0 is the "dimensionless length"; c is the coefficient of the tube time constant; p is the Laplace transformation with respect to the variable τ; ω is the cyclic frequency, 1/sec.

REFERENCES

1. Ya. Takahasi, Proc. Cranfield Conf. on Automatic Control, 1951 [Russian translation], IL, 1954.

2. B. G. Volik, Avtomatika i telemekhanika, 26, no. 3, 1965.

3. J. W. Rizika, Trans. ASME, 76, no. 3, 411, 1954.

18 April 1966

Institute for the Automation of Enterprises in the Building Materials Industry, Leningrad